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REVIEW ARTICLE

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Electromagnetic momentum in frontiers of modern physics

Abstract We review the role of the momentum of the electromagnetic (EM) fields $\boldsymbol{P}_e$ in several areas of modern physics. $\boldsymbol{P}_e$ represents the EM interaction in equations for matter and light waves propagation. As an application of wave propagation properties, a first order optical experiment which tests the speed of light in moving rarefied gases is presented. Within a classical context, the momentum $\boldsymbol{P}_e$ appears also in proposed tests of EM interactions involving open currents and angular momentum conservation laws.

Moreover, $\boldsymbol{P}_e$ is the link to the unitary vision of the quantum effects of the Aharonov-Bohm (AB) type and, for several of these effects, the strength of $\boldsymbol{P}_e$ is evaluated. These effects provide a quantum approach to evaluate the limit of the photon mass $m_{\rm ph}$. A new effect of the AB type, together with the scalar AB effect, provides the basis for table-top experiments which yield the limit $m_{\rm ph} = 9.4 \times 10^{-52}$g, a value that improves the results achieved with recent classical and quantum approaches.

Keywords electromagnetic momentum, Aharonov-Bohm effect, magnetic model of light, Rowland experiment, photon mass

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1 Introduction

Electromagnetic (EM) momentum is a classical physical quantity that appears in the standard description of the energy and momentum of EM fields, components of the EM tensor $\Theta^{\alpha\beta}$. We are considering here the *interaction* EM momentum $\boldsymbol{P}_e$, a quantity that is attracting physicists' attention as it arises in different scenarios of modern physics involving EM interactions. One of these scenarios is that of light propagation in slowly moving media [1–4]. Another is that of a unitary view of quantum nonlocal effects of the Aharonov-Bohm (AB) type [5–12]. More commonly, the interaction em momentum $\boldsymbol{P}_e$ appears as a nonvanishing quantity in EM experiments involving "open" or convection currents, while $\boldsymbol{P}_e$ vanishes in the common em experiments or interactions with closed currents or circuits [3, 4, 13, 14].

The main purpose of this article is to review the recent advances of physics involving the EM momentum $\boldsymbol{P}_e$ and its role in the proposal of new tests or in making other advances, such as setting a new limit on the photon mass.

In the field of electromagnetism, a growing number of articles questioning the standard interpretation of special relativity have now appeared [15–25]. Some of the authors adhere to a point of view close to the historical works of Lorentz and Poincaré, who maintained the existence of a preferred frame. For example, Selleri [17, 18] has developed Bell's idea [15, 16] and obtained a true Lorentzian theory, fully compatible with Einstein's special relativity in its description of physical phenomena, but very different in interpretation and philosophy. Another Lorentzian theory, very close to Selleri's formulation, was derived independently in Refs. [19–21].

It has been argued that these different formulations of Special Relativity are truly compatible only in vacuum, as differences may appear when light propagates in transparent moving media. Thus, Consoli and Costanzo [22–26], Cahill and Kitto [27, 28], and Guerra and de Abreu [19–21], point out that, for the experiments of the Michelson-Morley type, which are often said to have given a null-result, this is not the case and cite the famous work by Miller [29]. The claim of these authors is that the available data point towards a consistency of non-null results when the interferometer is operated in the "gas-mode", corresponding to light propagating through a gas [22–26] (as in the case of air or helium, for instance, even in modern maser versions of optical tests [30–32]).

Moreover, tests that involve EM interactions in open currents or circuits have been reconsidered by Indorato and Masotto [33] who point out that these experiments are not completely reliable and may be inconclusive [3, 4]. Because of all this, physicists have recently proposed experiments about those predictions of the theory that have not been fully tested, or they have formulated untested assumptions that differ from the standard interpretation of Special Relativity [3, 4, 13, 14, 17–26].

The interesting point is that all the above-mentioned scenarios and polemical hypotheses are linked to the interaction EM momentum. Therefore, throughout this article we highlight the role of $\boldsymbol{P}_e$ in each one of these scenarios. In doing this we first link the wave equation for light propagation in moving media to the wave equation for matter waves in quantum effects of the AB type. This analogy leads to the proposal of new optical experiments which test the recent controversial hypothesis [22–26] of light propagation in rarefied moving media. Within classical electrodynamics we point out a new dedicated experiment that could feasibly test the nonconservation of the angular momentum of an isolated system in order to indirectly prove the reality of $\boldsymbol{P}_e$, a test that has never been performed and could settle the objections raised in Ref. [33].

Finally, we consider the unitary view of the effects of the AB type in terms of the interaction em momentum, with the aim of finding advanced applications to electrodynamics. This unitary view [10–12] has recently led to the discovery of a new quantum effect and the related table-top experiment that yields relevant improved values of the photon mass limit [72]. Using this quantum approach and by means of a table-top experiment that exploits the scalar AB effect, we determine the improved photon mass limit $m_\gamma = 10^{-52}$g, which is so far the best limit obtained by either a classical or quantum approach.

2 Untested aspects of electromagnetism

2.1 Wave equations for matter and light waves

To elucidate the role of EM momentum in modern physics, we start by considering the wave equations for matter and light waves and show how the interaction term $\boldsymbol{Q}$ of these equations is related to $\boldsymbol{P}_e$ [36]. In general, with T^M_{ik} the Maxwell stress-tensor, the covariant description of the em momentum leads to the four-vector em momentum P^α_e expressed as

$$P^i_e\, c = \gamma \int (c\boldsymbol{g} + T^M_{ik}\beta^i)\mathrm{d}^3\sigma$$
$$cP^0_e = \gamma \int (u_{\mathrm{EM}} - \boldsymbol{v}\cdot\boldsymbol{g})\mathrm{d}^3\sigma \tag{1}$$

where $\beta = v/c$, and the em energy and momentum are evaluated in a special frame $K^{(0)}$ moving with velocity $\boldsymbol{v}$ with respect to the laboratory frame. Here, u_{EM} is the energy density and $\boldsymbol{S} = \boldsymbol{g}c$ is the energy flux or flow.

The analogy between the wave equation for light in moving media and that for charged matter waves has been pointed out by Hannay [1] and later addressed by Cook, Fearn and Milonni [2] who have suggested that light propagation at a fluid vortex is analogous to the Aharonov-Bohm (AB) effect, where charged matter waves (electrons) encircle a localized magnetic flux [5–9]. Generally, in quantum effects of the AB type [5–12] matter waves undergo an em interaction as if they were propagating in a flow of em origin that acts as a moving medium [10–12] and modifies the wave velocity. This analogy has led to the formulation of the so-called *magnetic model of light propagation* [1–4].

According to Fresnel [34], light waves propagating in a transparent, incompressible moving medium with uniform refraction index n, are dragged by the medium and develop an interference structure that depends on the velocity $\boldsymbol{u}$ of the fluid ($u \ll c$). At the time of Fresnel the preferred inertial frame was that at rest with the so-called ether, which here may be taken to coincide with the laboratory frame. The speed achieved in the ether frame is

$$v = \frac{c}{n} + \left(1 - \frac{1}{n^2}\right) u \tag{2}$$

as later corroborated by Fizeau [35]. Because of the formal analogy between the wave equation for light in slowly moving media and the Schrödinger equation for charged matter waves in the presence of the external vector potential $\boldsymbol{A}$ (i.e., the magnetic Aharonov-Bohm effect), both equations contain a term that is generically referred to as the interaction momentum $\boldsymbol{Q}$. Thus, the

Schrödinger equation for quantum effects of the AB type (with $\hbar = 1$) [10–12] and the wave equation for light in moving media can be written as [1–4]:

$$(-\mathrm{i}\boldsymbol{\nabla} - \boldsymbol{Q})^2 \Psi = p^2 \Psi \tag{3}$$

Eq. (3) describes matter waves if the momentum p is that of a material particle, while, if p is taken to be the momentum $\hbar k$ of light (in units of $\hbar = 1$), Eq. (3) describes light waves.

(a) All the effects of the AB type discussed in the literature [5–12] can be described by Eq. (3), provided that the interaction momentum $\boldsymbol{Q}$ is related [10–12, 36] to $\boldsymbol{P}_e$, the momentum of the EM fields. The AB term $\boldsymbol{Q} = (e/c)\boldsymbol{A}$ of the magnetic AB effect is obtained by taking $\boldsymbol{Q} = \boldsymbol{P}_e = \dfrac{1}{4\pi c}\int(\boldsymbol{E} \times \boldsymbol{B})\mathrm{d}^3\boldsymbol{x}'$ where $\boldsymbol{E}$ is the electric field of the charge and $\boldsymbol{B}$ the magnetic field of the solenoid. A general proof that this result holds in the *natural* Coulomb gauge, has been given by several authors [10–12, 38, 39]. For these quantum effects, the solution to Eq. (3) is given by the matter wave function:

$$\Psi = \mathrm{e}^{\mathrm{i}\phi}\Psi_0 = \mathrm{e}^{\mathrm{i}\int \boldsymbol{Q}\cdot \mathrm{d}\boldsymbol{x}}\Psi_0 = \mathrm{e}^{\mathrm{i}\int \boldsymbol{Q}\cdot \mathrm{d}\boldsymbol{x}}\mathrm{e}^{\mathrm{i}(\boldsymbol{p}\cdot\boldsymbol{x} - Et)}\mathcal{A} \tag{4}$$

where Ψ_0 solves the Schrödinger equation with $\boldsymbol{Q} = 0$.

(b) Calculations of the quantity $\boldsymbol{Q} = \boldsymbol{P}_e$ (1) for light in slowly moving media show [36] that the interaction term yields the Fresnel-Fizeau momentum [3, 4]

$$\boldsymbol{Q} = -\frac{\omega}{c^2}(n^2 - 1)\boldsymbol{u} \tag{5}$$

and that a solution of the type described in Eq. (4) may assume the forms

$$\begin{aligned}\Psi &= \mathrm{e}^{\mathrm{i}\phi}\Psi_0 = \mathrm{e}^{\mathrm{i}\int \boldsymbol{Q}\cdot \mathrm{d}\boldsymbol{x}}\mathrm{e}^{\mathrm{i}\int(\boldsymbol{k}\cdot \mathrm{d}\boldsymbol{x} - \omega \mathrm{d}t)}\mathcal{A}\\ \Psi &= \mathrm{e}^{\mathrm{i}\int(\boldsymbol{K}(\boldsymbol{x})\cdot \mathrm{d}\boldsymbol{x} - \omega \mathrm{d}t)}\mathcal{A}\end{aligned} \tag{6}$$

where $\boldsymbol{k}$ and $\boldsymbol{K}(\boldsymbol{x})$ are wave vectors, $\omega = k\,c/n$ the angular frequency, and n the index of refraction, while Ψ_0 solves Eq. (3) with $\boldsymbol{Q} = \boldsymbol{u} = 0$. Actually, the wave equation (3) can be derived without reference to special relativity by taking into account the polarization of the moving medium [37].

The fact that the interaction momentum $\boldsymbol{Q}$ is related to $\boldsymbol{P}_e$ [10–12, 36] for both matter waves of effects of the AB type [10–12] and light waves in moving media [36], definitely reinforces the existing analogy between the two wave equations. Two theoretical possibilities arise [3, 4]:

— By incorporating the phase ϕ in the term $\int \boldsymbol{K}(\boldsymbol{x})\cdot \mathrm{d}\boldsymbol{x}$, the last expression on the rhs of Eq. (6) keeps the usual invariant form of the solution as required by special relativity and one finds [36] for the speed of light the result $\boldsymbol{v} = (c/n)\widehat{\boldsymbol{c}} + (1 - 1/n^2)\boldsymbol{u} = (c/n)\widehat{\boldsymbol{c}} - \boldsymbol{Q}(c^2/n^2\omega)$ in agreement with Eq. (2) and Special Relativity.

— Maintaining instead the analogy with the AB effect, the solution can be chosen to be represented by the first term of Eq. (6), $\Psi = \mathrm{e}^{\mathrm{i}\phi}\Psi_0$. In this case, the phase velocity changes but the speed of light (the particle, or photon) may not change [3, 4]. This result is in total agreement with the analogous result for the AB effect where $\boldsymbol{Q} = (e/c)\boldsymbol{A}$ and the particle speed is left unchanged by the interaction with the vector potential $\boldsymbol{A}$.

We consider now more in detail possible consequences of the untested second theoretical possibility, which implies that the phase velocity of light does not coincide with that of the particle (photon). If the magnetic model of light propagation in moving media holds, as happens in AB effects, the additional Fresnel-Fizeau momentum $(n^2 - 1)\omega\boldsymbol{u}/c^2$ is carried by the medium and not by the light particles. That the photon momentum does not change is assured by the fact that the relationship $\omega = kc/n$ still holds and the group velocity is given by $\mathrm{d}\omega/\mathrm{d}\boldsymbol{k} = \boldsymbol{c}/n$ and no longer by Eq. (2).

From an experimental or phenomenological point of view, the only thing that can be concluded from the outcome of the above mentioned interferometric tests, and in particular from that of the Fizeau experiment itself, is that the phase and the speed of the waves are modified by the flow $\boldsymbol{Q} \propto \boldsymbol{u}$. However, these interferometric tests are unable to measure the group velocity, the energy and the momentum of the particles involved. This implies that all the Fizeau-type experiments based on interferometric techniques do not represent a conclusive test or a confirmation of the relativistic addition of velocities for particles. Therefore, the non-interferometric approach to the measurement of the speed of photons in moving media that we consider below, possesses a physical relevance not contained in the traditional Fizeau approach.

In order to test the magnetic model of light, we describe qualitatively an experiment capable of measuring the speed of photons in a moving medium. We are interested in showing that such an experiment is feasible and can be realized with present technology. Technical aspects and details of a proposed experiment of this type are given elsewhere.

This method uses a device D_{S} that acts as a source capable of emitting a short burst of photons or pulse of light of duration τ, as shown in Fig. 1. The pulse travels through a pipe inside which a moving fluid propagates and, as it exits, hits a photodetector D_{ph} that is electrically connected to the source D_{S}. When the photodetector D_{ph} is hit by the pulse of light it triggers the source D_{S} which emits a second light pulse that makes another trip through the pipe. Furthermore, this apparatus triggers circuitry to record the number of times N a light pulse has completed a trip. Let us denote by t_D

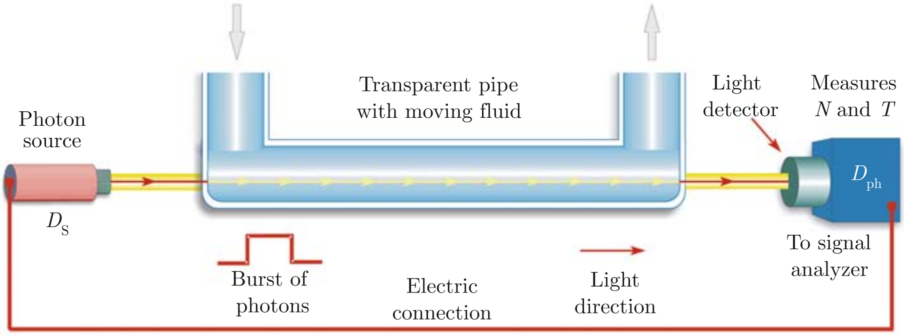

Fig. 1 A short burst of light emitted by the source D_S propagates through a transparent fluid moving with velocity u in a pipe of length L and then hits a photodetector D_{ph}. At this time, the device D_{ph}, which is electrically connected to the source D_S, triggers it so that another burst of light is emitted, and so on. The device D_{ph} records the number of times N a burst of photons strikes it and the total time T taken by the iterative process. Knowing the other parameters, the speed of photons is determined by measuring N and T.

the time delay between the moment the light pulse hits D_{ph} and the moment D_S emits a new light pulse after being triggered by D_{ph}. A clock connected to D_{ph} measures the total time T elapsed after N trips, which is given by

$$T(u) = \frac{NL}{c/n(u)} + Nt_D \tag{7}$$

The time $T(u)$ can be compared with the time T measured in the same conditions but for the medium at rest to yield

$$T(u) - T = \frac{NL}{c}[n(u) - n] \tag{8}$$

With $n(u) = n[1 + (c/nNL)(T(u) - T)]$ from Eq. (8), the resulting measured speed difference is

$$\frac{c}{n(u)} - \frac{c}{n} = \frac{c^2}{n^2 N L}[T - T(u)] \tag{9}$$

which can be determined by measuring N and $T(u) - T$.

As shown in Refs. [3, 4], this experiment is within the reach of present technology and, thus, is capable to test the magnetic model of light and determine if the speed of light waves in slowly moving media differs from the speed of light particles (photons).

As another application of the magnetic model of light we consider in the next section the case of light propagation in highly rarefied moving media, an aspect of light propagation that has been recently discussed in the literature by Consoli and Costanzo [22–26].

The established relation (5) will be used in the next sections to tentatively express in a quantitative way the hypothesis of Consoli and Costanzo [22–26] referring to v, the speed of light in a moving rarefied media. With a quantitative expression for v it is then possible to formulate a dedicated experiment that tests Consoli and Costanzo's hypothesis.

2.2 Propagation of EM waves in rarefied moving media

Duffy [57] has noted that the concept of an ether-like preferred frame has always incited controversy, even in modern scientific investigations aimed at exploring the less understood aspects of relativity theory. Within this scenario, Consoli and Costanzo [22–26], Cahill and Kitto [27, 28], and Guerra and de Abreu [19–21], after a reanalysis of the optical experiments of the Michelson–Morley type, claim that the available data point towards a consistency of non-null results when light in the arms of the interferometer propagates in a rarefied gas, like the cases of air at normal pressure and temperature. The possibility of maintaining the existence of a preferred frame, and parallel interests in the Michelson-Morley, Trouton-Noble and related effects, arises because the coordinate transformation used, the Tangherlini transformations [41–44] (denoted by *inertial transformation* in Refs. [17, 18] and by *synchronized transformation* in Refs. [19–21]), foresee the same length contraction and time dilation of the Lorentz transformations. However, they contain an arbitrariness in the determination of the time synchronization parameter, with the consequence that there are quantities which eventually cannot be measured, such as the one-way speed of light, its measured value depending on the synchronization procedure adopted [41–44]. Different synchronization procedures are possible [15–26], fully compatible with Einstein's relativity in practice, but with very different assertions in fundamental and philosophical terms [17, 18].

The original important assumption made by Consoli *et al.* to corroborate their claims of a non-null result and open a window for the possible existence of a preferred frame, is that light in a moving rarefied gas of refractive index n very close to 1 propagates with speed c/n, isotropically, in the preferred frame, as if the medium were not moving. Obviously, this hypothesis is in contrast with special relativity that foresees the speed (2), but it is not ruled out by the known optical tests. Thus, this assumption needs justification and experimental corroboration.

In the following, we explore possible modifications of the form of the present Fresnel-Fizeau momentum when the moving medium is composed of rarefied gas. For simplicity, we suppose that the light phase and particle velocities in moving media coincide. In the case of light propagation it is the interaction fields that characterizes the flow $\boldsymbol{Q} \div \boldsymbol{u}$ and the corresponding light dragging or light delay, so that the velocity of photons through a moving medium will be affected to a degree that depends on the various properties of the interaction fields in the polarizable medium, such as the intensity and relative spatial extension over the total volume, both measures of the effective local-interaction EM energy.

It is not unconceivable that the effectiveness of the light delay mechanism in a compact moving medium differs, and perhaps even substantially so, from that of a non-compact moving medium, such as a rarefied gas, even if they have the same index n. As an *ad hoc* hypothesis or a tentative model of a light delay mechanism, it has been supposed [76] that its effectiveness e_f arises from the relative spatial extension V_i of the interaction EM momentum $\boldsymbol{Q}(\boldsymbol{u})$ with respect to the extension V of the total EM momentum. Introducing then the ratio $e_f = V_i/V$, the effective EM interaction momentum, to be used in determining the speed of light in a moving media, will be assumed to be given by the effective Fresnel-Fizeau term $e_f\boldsymbol{Q} = (V_i/V)\boldsymbol{Q}$, while the resulting velocity of light in moving rarefied media is

$$\boldsymbol{v} = \frac{c}{n}\widehat{\boldsymbol{c}} - \frac{c^2}{n^2\omega}e_f\boldsymbol{Q} = \frac{c}{n}\widehat{\boldsymbol{c}} + e_f\left(1 - \frac{1}{n^2}\right)\boldsymbol{u} \tag{10}$$

The hypothesis of Consoli *et al.* of the speed c/n in the preferred frame for moving rarefied gases, will be justified by our model if $e_f = V_i/V$ turns out to be very small and, in this case, negligible. Calculations leading to a rough estimate of V_i/V for air at room temperature yield [76] $e_f = N_a(a^3/R^3)\,22.9 = 6.1 \times 10^{-3}$, which indeed can be neglected. Thus, our model foresees that the speed of light in moving media is actually not c/n but, quantitatively, the changes found do not alter significantly the basic hypothesis and resulting analysis by Consoli *et al.* [22–26] and Guerra *et al.* [19–21].

Recall that other approaches outside classical electrodynamics suggest the validity of the above assumption [22–26]. As mentioned by Consoli *et al.* one argument is based on the presence of ingredients that are often found in present-day elementary particle physics, namely: (1) vacuum condensation, as with the Higgs field in the electroweak theory, and (2) an approximate form of locality, as with cutoff-dependent, effective quantum field theories [45]. The resulting picture is closer to a medium with a non-trivial refractive index [22–26] than to the empty space-time of special relativity.

3 Optical test in the first order in v/c

The main consequence is that, with the present hypothesis of negligible drag-like effect for moving rarefied gases, ether drift experiments of the order v/c become meaningful again. Let us consider for example the following experiment which is a variant of the Mascart and Jamin experiment of 1874 [40].

A ray of light travels from point A to point B of a segment A–===–B representing an optical interferometer. The original ray is split into two rays at A, which propagate separately through the two arms (1 and 2) of the interferometer. The rays recombine then at B where the interference pattern is observed. The arms 1 and 2 are made of a transparent rarefied gases or materials with indices of refraction n_1 and n_2 and wherein the speeds are c/n_1 and c/n_2 in the preferred frame, respectively, in agreement with Consoli's *et al.* hypothesis [22–26] of the velocity expression (10) with $e_f = 0$. The laboratory frame with the interferometer and the rarefied gas is moving with speed u with respect to the preferred frame. We could be using the expressions for the speed in the moving laboratory frame resulting from the Tangherlini transformation, which can be found in Refs. [19–21, 41–44]. The calculation can also be done using the standard velocity addition from the Lorentz transformation, i.e., using the definition of *Einstein speed* as detailed in Refs. [19–21]. Both approaches yield the same result. The speed of light in arm 1 in the frame of the interferometer, moving with speed u with respect to the preferred frame, is respectively

$$w_1 = \frac{c/n_1 - u}{1 - u^2/c^2} \quad \text{or} \quad w_1 = \frac{c/n_1 - u}{1 - u/(c\,n_1)} \tag{11}$$

and analogously for w_2. If L is the length of the arms, the time delay, or optical path difference, for the two rays yields, in the first order in u/c,

$$\begin{aligned}\Delta t(0^\circ) &= L\left(\frac{1}{w_1} - \frac{1}{w_2}\right)\\ &\approx \frac{L}{c}(n_1 - n_2)\left[1 + \frac{u}{c}(n_1 + n_2)\right]\end{aligned} \tag{12}$$

In order to observe a fringe shift, the interferometer needs to be rotated, typically by 90° or 180°. The time delay for 180° is the same of Eq. (12) with u replaced by $-u$. The observable fringe shift upon rotation of the interferometer does not vanish in the first order in u/c and is related to the time delay variation

$$\begin{aligned}\delta t &= \Delta t(0^\circ) - \Delta t(180^\circ)\\ &\approx 2\frac{u}{c}(n_1^2 - n_2^2)\frac{L}{c}\end{aligned} \tag{13}$$

Choosing two media with different refractive index such that $n_1^2 - n_2^2$ is not too small ($> 10^{-3}$), the resulting fringe shift should be easily observable if the preferred frame exists and its speed u is not too small. Knowing the sensitivity of the apparatus, one could set the lower limit of the observable preferred speed u. Interferometers, used in advanced Michelson-Morley's type of experiments, could detect a speed u as small as 1 km/s (a few m/s for He-Ne maser tests [32]). Thus, this optical experiment, in passing from second order (u^2/c^2) to first order tests, should be able to improve the range of detectability of u by a factor $(c/u)(n_1^2 - n_2^2) \approx 3\times10^5\times10^{-3} = 3\times10^2$, i.e., detect with the same interferometer speeds 3×10^2 smaller.

New, more refined versions of the Michelson-Morley type of experiment (including tests using He-Ne masers.) are not suitable to test the hypothesis of Consoli *et al.* [22–26] because of the relatively low sensitivity of these experimental approaches for rarefied gases. However, as shown above, an optical test in the first order in v/c becomes meaningful in this case and can provide important advantages over the second order experiments of the Michelson-Morley type.

4 Untested EM effects of open currents

We consider tests related to investigations of Rowland's effect [46], viz., that electrified bodies in motion produce magnetic effects. Rowland's experiment consisted in detecting the magnetic field produced by moving charges. It is generally claimed that the outcome of Rowland's experiment of 1876 indicates that isolated moving charges produce a magnetic field. In collaboration with von Helmholtz, Rowland performed the experiment using a circular parallel plate capacitor that was charged and set spinning about its axis of symmetry. The resulting magnetic field was found to affect a delicately suspended pair of astatic magnetic needles hung in proximity to the disk just as would, by Oersted's rule, a circular electric current coincident with the periphery of the disk.

The experiments were repeated and confirmed by Röntgen [47, 48] and by Himstedt [49]. Later Crémieu again repeated them and obtained negative results [50–53] that were confirmed by Korda. Later, they were again reconducted by Pender [54] and by Adams [55]. The publication of Crémieu's experiments raised a controversy that involved leading French and British physicists, as pointed out in a review paper by Indorato and Masotto [33]. Poincaré was the mentor of Crémieu at the time and apparently was the one who suggested what kind of experiment to perform. Pender was Rowland's student and was mentored by him until his death in 1901.

In order to understand the difficulties involved in this type of experiment, it should be pointed out that in the attempts made by Rowland to observe the magnetization produced by moving electrified bodies, the magnetic field to be detected was about 1/50000 times the horizontal component the Earth's magnetic field, a very small value unlikely to be observable.

The results of Crémieu and Pender's joint, late experiments seemed to favor the existence of Rowland's effect. However, the authors were cautious in their final comments [56]:

"*It is not for us to say if these effects are really due to electric convection in the sense in which Faraday and Maxwell understood this expression, nor to decide if they are in accord with the fundamental hypotheses of present theories.*"

Rowland's experiment involves fields or forces of open currents, i.e., moving charged objects. In dual Rowland's experiments, the charged objects are at rest in the laboratory reference frame, as shown in Fig. 2. A moving observer would detect a magnetic field produced by the relatively moving charged objects.

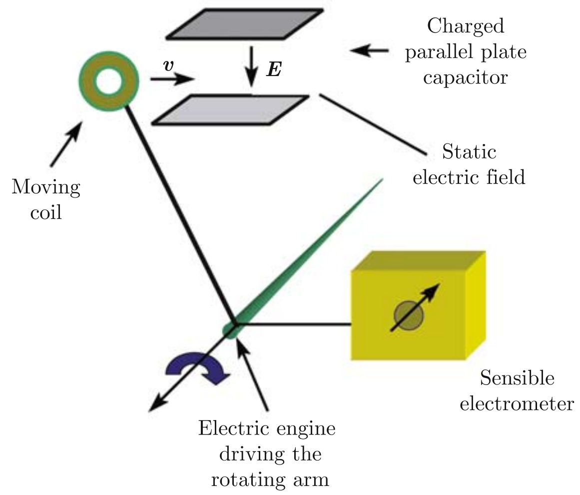

Fig. 2 An example of the untested dual Rowland's experiment. A static electric field $\boldsymbol{E}$ is capable of producing a time-varying magnetic induction in a moving coil. Detection of the induced current or emf in the coil would be an experimental proof of the validity of the dual Rowland's effect.

A charge at rest does not interact with a neutral circuit carrying an electric current unless the current varies with time. In this case, the force involved in related tests that acts on "open" charges at rest is due to the time varying electric field $\boldsymbol{E} = -c^{-1}\partial_t \boldsymbol{A}$ where $\boldsymbol{A}(t)$ is the vector potential.

The force on a charge q is then $q\boldsymbol{E} = -qc^{-1}\partial_t \boldsymbol{A}$, but it turns out that $qc^{-1}\boldsymbol{A}$ also represents the Aharonov-Bohm interaction term that in previous sections has been linked to the interaction em momentum $\boldsymbol{P}_e$. With available charge and current distributions, the force $-qc^{-1}\partial_t \boldsymbol{A}$ is very small and has never been tested directly with open currents or charges. Because of the link between this force and $\boldsymbol{P}_e$, a test of the force represents, indirectly, a test of the reality of the interaction em momentum $\boldsymbol{P}_e$. Considering that historical tests on open currents are not completely reliable and that even optical tests are inconclusive, it is worthwhile to consider here other tests of electromagnetism the outcome of which may be useful to dispel doubts and settle controversial points.

For this purpose we consider here a test of violation of the mechanical angular momentum of an isolated system. Consider a cylindrical capacitor of inner radius a, outer radius b, and length L. The capacitor is suspended vertically along its symmetry axis and placed in a uniform magnetic field $\boldsymbol{B}$, which is also directed vertically. When the capacitor is charged with a potential difference V and the magnetic field varies with time, a charge q at the distance r from the axis experiences a torque $\tau = rq\partial_t A$ due to the action of the force $-q\partial_t \boldsymbol{A}$ where $\boldsymbol{A}$ is the time varying vector potential. For a charge distribution λ on the cylinder the application of Stokes' theorem leads to the corresponding angular impulse

$$\int \tau \mathrm{d}t = r\lambda \oint \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{l} = r\lambda BS = \pi\lambda B r^2 \tag{14}$$

Since the the charge on the inner and outer surfaces is the same, the total angular impulse on the capacitor is $\varDelta = \int \tau_b \mathrm{d}t - \int \tau_a \mathrm{d}t = q\pi B(b^2 - a^2)/2$. This impulse transfers an angular velocity ω to the capacitor of mass M such that $\varDelta = I\omega$ where $I \approx Mb^2$ is the moment of inertia. If k is the torsion constant and θ the maximum angular displacement, on account of the relation $(1/2)I\omega^2 = (1/2)k\theta^2$ we may write

$$\theta = \sqrt{\frac{1}{kM}} \frac{q}{2} B(b^2 - a^2) \tag{15}$$

The total charge q is given by $q = VC = 2\pi\epsilon\varepsilon_o LV/\ln(b/a)$ where C is the capacitance, and ϵ and ε_o the relative and the dielectric constant respectively. There are dielectric materials with very high relative dielectric constant ($\epsilon \approx 10^4$) and, with $V = 10^4$ V, in a long capacitor ($L = 1$ m, $b = 3a = 5$ cm) q can reach values of the order of 5×10^{-3} C. With the torsion constant $k = 10^{-8}$ Nm/rad, $M = 10^{-1}$ kg and $B = 10^{-1}$ T, we obtain $\theta \approx 2$ rad, which is a very large angle. Therefore, in the actual experiment one could reduce the potential difference V or use less exotic dielectric materials with lower values of ϵ.

It is straightforward to evaluate the em angular momentum $\boldsymbol{N}_{\mathrm{em}} = \int \boldsymbol{r} \times (\boldsymbol{E} \times \boldsymbol{B})\mathrm{d}V$ in this configuration. Since $E = \lambda/(2r)$ is radial and B is uniform, the em angular momentum yields $q\pi B(b^2 - a^2)/2$ which corresponds to the angular impulse $\varDelta$ on the capacitor. As expected, we have $\tau = -(\mathrm{d}N_{\mathrm{EM}}/\mathrm{d}t)$. Thus, the em momentum stored in the capacitor produces a torque when the field $\boldsymbol{B}$ varies with time.

The conservation law for the total angular momentum $(\boldsymbol{N}_{\mathrm{EM}} + \boldsymbol{N}_{\mathrm{mech}})$ is as follows. Before $\boldsymbol{B}$ is switched off there is a nonvanishing em momentum and corresponding angular momentum $\boldsymbol{N}_{\mathrm{EM}}$, while the mechanical momentum $\boldsymbol{N}_{\mathrm{mech}}$ is zero. By switching off the field $\boldsymbol{B}$, $\boldsymbol{N}_{\mathrm{EM}}$ vanishes but $\boldsymbol{N}_{\mathrm{mech}}$ is produced, so that the total angular momentum is conserved. Assuming, as is well-known, that there is no reaction on the sources of $\boldsymbol{B}$, the outcome of the experiment represents a proof of the violation of the mechanical angular momentum of an isolated system, a dynamic feature that has never been tested directly.

5 Effects of the Aharonov-Bohm type and the photon mass

We have shown in the previous sections that all the effects of the AB type can be described in a unified way by the wave equation (3) where, for each one of the effects, the quantity $\boldsymbol{Q}$ represents the em interaction momentum (1). Both the interaction energy and momentum appear in the expression of the phase of the quantum wave function. Through the phenomenon of interference, phase variations can be measured and the observable quantity can be related to variations of the interaction em momentum or energy. In the following sections we show how the photon mass can be determined by measuring its effect on the observable phase variation via the related changes of em momentum or energy.

The possibility that the photon possesses a finite mass and its physical implications have been discussed theoretically and investigated experimentally by several researchers [58–64]. Originally, the finite photon mass m_γ (measured in cm^{-1}) has been related to the range of validity of Coulomb law [58–62]. If $m_\gamma \neq 0$ this law is modified by the Yukawa potential $U(r) = \mathrm{e}^{-m_\gamma r}/r$, with

$m_\gamma^{-1} = \hbar/(m_{\rm ph}c) = \lambda_{\rm C}/(2\pi)$ where $m_{\rm ph}$ is expressed in *grams* and $\lambda_{\rm C}$ is the Compton wavelength of the photon.

There are direct and indirect tests for the photon mass, most of them based on classical approaches. Recalling some of the classical tests, we mention the results of Williams *el al.* and of various other workers [58–62] yielding the range of the photon rest mass $m_\gamma^{-1} > 3\times 10^9$cm, and of Luo, Tu, Hu, and Luan [63, 64] yielding the range $m_\gamma^{-1} > 1.66\times 10^{13}$cm and corresponding photon mass $m_{\rm ph} < 2.1\times 10^{-51}$g.

Several conjectures related to the Aharonov-Bohm (AB) effect [5–9] have been developed assuming electromagnetic interaction of fields of infinite range, i.e., zero photon mass. The possibility that any associated effects become manifest within the context of finite-range electrodynamics has been discussed by Boulware and Deser (BD) [65]. In their approach, BD consider the coupling of the photon mass m_γ, as predicted by the Proca equation $\partial_\nu F^{\mu\nu} + m_\gamma^2 A^\mu = J^\mu$, and calculate the resulting magnetic field $\boldsymbol{B} = \boldsymbol{B}_0 + \widehat{\boldsymbol{k}} m_\gamma^2 \Pi(\rho)$, that might be used in a test of the AB effect. Because of the extra mass-dependent term, BD obtained a nontrivial limit on the range of the transverse photon from a table-top experiment yielding $m_\gamma^{-1} > 1.4\times 10^7$cm.

After the AB effect, other quantum effects of this type have been developed, such as those associated with neutral particles that have an intrinsic magnetic [66] or electric dipole moment [10–12], and those with particles possessing opposite electromagnetic properties, such as opposite dipole moments or charges [10–12, 67–71]. The impact of some of these new effects on the photon mass has been studied by Spavieri and Rodriguez (SR) [72].

Based on theoretical arguments of gauge invariance, SR point out that, in analogy with the AC effect for a coherent superposition of beams of magnetic dipoles of opposite magnetic moments $\pm\mu$ [68–70] and the effect for electric dipoles of opposite moments $\pm d$ [71], the Spavieri effect [67] of the AB type for a coherent superposition of beams of charged particles with opposite charge state $\pm q$ is theoretically feasible. Using this effect, SR evaluate its relevance in eventually determining a bound for the photon mass $m_{\rm ph}$. SR consider a coherent superposition of beams of charged particles with opposite charge state $\pm q$ passing near a huge superconducting cyclotron. The $\pm$ charges feel the effect of the vector potential $\boldsymbol{A}$ created by the intense magnetic field of the cyclotron and the phases of the associated wave function are shifted, leading to an observable phase shift [72]. For a cyclotron of standard size, SR show that the limit

$$m_\gamma^{-1} = 10^6\, m_{\gamma{\rm BD}}^{-1} \approx 2\times 10^{13}{\rm cm}$$

is achievable. With their table-top experiment, BD obtained the value $m_{\gamma{\rm BD}}^{-1} \approx 140$ km that is equivalent to $m_{\rm phBD} = 2.5\times 10^{-45}$g. With SR approach, the new limit of the photon mass is $m_{\rm ph} \approx 2\times 10^{-51}$g which is of the same order of magnitude of that found by Luo *et al.* [63, 64]. Of course, by increasing the size of the cyclotron a better limit could be obtained. With the standard technology available, we expect that the limit $m_{\rm ph} \approx 2\times 10^{-52}$g is not out of reach.

5.1 The scalar Aharonov-Bohm effect and the photon mass

Having exploited the magnetic AB effect in the previous section, we consider now the scalar AB effect. In this effect charged particles interact with an external scalar potential V. The standard phase φ_s acquired during the time of interaction is $\varphi_s = \frac{1}{\hbar}\int eV(t)\,{\rm d}t$.

In the actual test of the scalar AB effect, a conducting cylinder of radius R is set at the potential V during a time τ while electrons travel inside it. Since no forces act on the charges it is a field-free quantum effect. If the photon mass does not vanish the potential is modified according to Proca equation. Gauss' law is modified and the potential Φ obeys the equation $\nabla^2\Phi - m_\gamma^2\Phi = 0$, with the boundary condition that the potential on the cylinder be V. In cylindrical coordinates the solutions are the modified Bessel functions of zero order, $I_0(m_\gamma\rho)$ and $K_0(m_\gamma\rho)$ which are regular at the origin and infinite, respectively. It follows that the acceptable solution is

$$\Phi(\rho) \approx V\left[1 + \frac{m_\gamma^2}{2}\left(\rho^2 - R^2\right)\right] \tag{16}$$

where the first two terms of the expansion of $I_0(m_\gamma\rho)$ have been considered [74].

For two interfering beams of charges passing through separate cylinders, the relative phase shift is

$$\delta\varphi_s = \frac{1}{\hbar}\int e\left[V_1(t) - V_2(t)\right]{\rm d}t \tag{17}$$

where $V_1(t)$ and $V_2(t)$ are the potentials applied to cylinder 1 and 2, respectively. Consequently, according to Eq. (16), the contribution of the photon mass to the relative phase shift is

$$\delta\varphi = \delta\varphi_s + \Delta\varphi = \delta\varphi_s + \frac{m_\gamma^2}{4}\left(\rho^2 - R^2\right)\delta\varphi_s \tag{18}$$

Obviously, this additional phase shift term vanishes if m_γ vanishes and the standard result is recovered. The last term of Eq. (18) is useful for determining the photon mass in a table-top experiment. We consider the simple case of one beam travelling inside cylinder 1 and the other travelling outside it ($V_2(t) = 0$) for a short time interval τ. It follows that $\Delta\varphi = \delta\varphi - \delta\varphi_s$ reads

$$\Delta\varphi = -\frac{em_\gamma^2}{4}\left(\rho^2 - R^2\right) V \frac{\tau}{\hbar} \tag{19}$$

where $V = V_1(t) - V_2(t)$. This is our main result for determining the photon mass limit. Interferometric experiments may be performed with a precision of up to 10^{-4}, therefore, following the approaches of BD and SR we set $\Delta\varphi = \varepsilon$, $\varepsilon = 10^{-4}$. Also, we suppose that the beam 1 travels nearly at the centre of the cylinder ($\rho \ll R$) so that

$$m_\gamma^{-1} = \frac{R}{2}\sqrt{\frac{\pi V\tau}{\varepsilon(h/2e)}} \tag{20}$$

The following values may be used to estimate m_γ^{-1}: $V = 10^7$V, $h/(2e) = 2.067 \times 10^{-15}\text{Tm}^2$, $\tau = 5 \times 10^{-2}$s and $R = 27$ cm. The corresponding range of the photon mass is

$$m_\gamma^{-1} = 3,4 \times 10^{13}\text{cm} \tag{21}$$

which yields the improved photon mass limit $m_{\text{ph}} = 9.4 \times 10^{-52}$g, but we are left to justify the values used above for τ and R, which are both quite high.

It is interesting to compare the strength of the AB phase of the scalar AB effect with that of the magnetic AB effect. The scalar AB phase may be expressed as $eV\tau/\hbar$, while the magnetic AB phase is $eAL/(c\hbar)$, and the link between the particle's classical path is $L = \tau v$ with v its speed assumed to be uniform. According to special relativity, magnetism is a second order effect of electricity, therefore in normal conditions the strength of the coupling eA/c is smaller than the coupling eV. As a consequence of this, the phase variation due to the finite photon mass should be smaller in the magnetic than in the scalar AB effect. In other words, the scalar AB effect should be yielding a better limit for the photon mass than the magnetic AB effect. However, the above consideration is valid if in the actual experiments we have comparable path lengths, i.e., if $\tau \approx L/v$. In the table-top experiment by SR [72] L is of the order of several meters. Choosing as charged particles heavy ions, for example $^{133}Cs^+$, their speed could be 27 m/s [75]. With this speed and $L = 1.35$ m for the cylinder length, we get $\tau = 5 \times 10^{-2}s$ for the time of flight inside the cylinder. Since $\tau \approx L/v$, the improved result (21) obtained by exploiting the scalar AB effect is justified.

However, the high values chosen for R and L imply that the charged particle beams will have to keep their state of coherence through an extended region of space $L = 1.35$ m during the interferometric measurement process, while in standard interferometry the path separations are of the order of at most a few *cm*. Thus, technological advances are needed in this respect, as also mentioned in the article by SR [72] and the references cited therein.

Nevertheless, the feasibility of testing the photon mass with the scalar AB effect has been confirmed by the recent work of Neyenhuis, Christensen, and Durfee [74], lending support to the quantum approach. Actually, it is conceivable the possibility of extending to the case of the scalar AB effect the techniques of Refs. [68–70] and [71] for a coherent superposition of beams of charged particles with opposite charge state $\pm q$, as suggested by SR in Ref. [72]. This may lead to achieve even better limits for the photon mass. This and other technical aspects of our table-top experimental approach will be elaborated elsewhere.

6 Conclusions

We have recalled that the interaction momenta $\boldsymbol{Q}$ of the effects of the AB type and of light in moving media have the same physical origin, i.e., are given by the variation of the momentum of the interaction EM fields $\boldsymbol{P}_e$. Expecting that the effectiveness of the light delay mechanism in a rarefied gas differs from that of a compact transparent fluid or solid, we consider a tentative model of light propagation that validates the analysis made by Consoli *et al.* [22–26] and Guerra *et al.* [19–21]. As a test of the speed of light in moving rarefied media and of the preferred frame velocity, we propose an improved first order optical experiment that is a variant of the historical Mascart-Jamin experiment.

The momentum of interaction em fields $\boldsymbol{P}_e$ appears also in tests of the force on open currents, while the reality of $\boldsymbol{P}_e$ is linked to tests of the violation of mechanical angular momentum. For this purpose we have proposed an experiment that evidences this violation for a suspended charged cylindrical capacitor in the presence of a time-varying $\boldsymbol{B}$ field.

Finally, we have considered the table-top approach of Boulware and Deser to the photon mass and verified its applicability to other effects of the AB type, concluding that the new effect using beams of charged particles with opposite charge state $\pm q$ for the magnetic AB effect, and the scalar AB effect are a good candidates for determining the limit of the photon mass. Using a quantum approach to evaluate the limit of m_{ph} with these effects, we perform realistic table-top experiments that yield the limit $m_{\text{ph}} = 9.4 \times 10^{-52}$g, an important result that either matches or improves the limits achieved with recent classical and quantum approaches. In conclusion, advances in this area indicate that quantum approaches the photon mass limit are feasible and may compete with and even surpass the traditional classical methods.

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